# Wide-Band Balanced Line Microwave Hybrids

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***Abstract*—Wide-band balanced line microwave hybrids each consisting of lossless two-ports and a "reversed-phase hybrid ring" are presented. The two-ports can be designed for wide-band match and the new device is shown to have the same frequency independent properties as the basic hybrid ring. Two special two-ports are treated: one consists of open-circuited series stubs and short-circuited shunt stubs while the other consists of transmission-line cascades. The hybrid ring with matching stubs has been described previously, but is difficult to realize. The hybrid ring matched with transmission-line cascades is a new development which simplifies the practical realization. A complete synthesis procedure is presented and design data for 3-, 6-, 10-, and 15-dB coupling are tabulated.**

## I. Introduction

There is no simple planar transmission-line realization of a 3-dB directional coupler with multioctave bandwidth. The theoretical performance of the coupled transmission-line (CTL) coupler [1], [2] is very impressive, but the realization is difficult. The CTL coupler has perfect match and directivity, but the coupling is frequency dependent. Tables of multisection CTL couplers are found in [2]. As an example, a seven-section 3-dB coupler with a coupling ripple of $\pm 0.05$ dB yields a bandwidth of 5.67 : 1. For the middle section of this coupler a normalized even-mode characteristic impedance of 4.40 $\Omega$ is required and the odd-mode characteristic impedance is 0.227 $\Omega$ (1-$\Omega$ terminations). A larger coupling ripple (and bandwidth) requires even tighter coupling in the middle section.

The branch-line (BL) coupler (branch guide (BG) for waveguide realization) [3] can also have wide-band performance. The BL coupler has no perfect properties, and it is difficult to achieve good match and coupling simultaneously. A synthesis procedure which optimizes match and isolation of BL couplers is given in [3], where tables are also found. As an example an eight-section 3-dB coupler with approximately equal VSWR ripple and VSWR $\leq 1.124$ (directivity $\geq 24.7$ dB) yields a bandwidth of 3 : 1. The coupling decreases monotonically from 3 dB at the center frequency to 0.25 dB (!) at the band edges. Very high (10.8 $\Omega$) and very low (0.324 $\Omega$) impedances are required for the realization of this coupler. The use of a nonoptimum design [4] reduces the maximum-to-minimum ratio of line impedances by a factor of approximately three. The realization is still very difficult, however, and the coupling varies considerably over the band.

The "reversed-phase hybrid ring" [5] is a device with some interesting properties. The name of the hybrid ring is due to a phase-reversal section in one of the branches. This hybrid ring is identical to the well-known "rat race" hybrid ring at the design frequency. The three-quarter-wave section of the "rat race" ring is replaced by a single quarter-wave section in cascade with a frequency-invariant phase shift of 180°. True frequency-independent 180° phase shift can be accomplished in twin line by simply twisting the two lines as Fig. 1 shows. Some theoretical curves and design formulas for this device are presented in Section II-B.

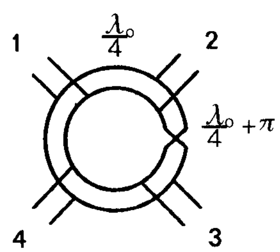


Fig. 1. The reversed-phase hybrid ring.

A coupler using the basic hybrid ring and general lossless reciprocal two-ports for match improvement is treated in Section II-A. The coupler presented by Podell [6] is a special case of the general coupler and is examined in Section II-C. The coupler presented in Section II-D is also a special case of the general coupler given in Section II-A. It is a new development in order to simplify the practical realization of a wide-band coupler. Cascades of transmission lines are used to achieve wide-band performance and the only difficulty is to realize the basic hybrid ring, but such realizations have been reported by Carr [7] for frequencies up to $X$ band and recently by Rubin and Saul [8] for the 40–60-GHz band.

## II. Theory

### *A. Lossless Reciprocal Two-Ports Improve the Match of the Basic Hybrid Ring*

Lossless reciprocal two-ports can be added to a reversed-phase hybrid ring without destroying the perfect properties of the original ring. The two-ports can then be designed to improve the match of the device. Fig. 2(a) shows how four equal two-ports are connected to the hybrid ring.

By splitting the 180° phase shifter into two 90° phase shifters it is possible to use even and odd modes for the analysis [9], [10]. The even-mode circuit is shown in Fig. 2(b). The even-mode reflection coefficient $\Gamma_e$ is seen from port 1, and, due to symmetry, the odd-mode reflection coefficient $\Gamma_o$ is seen from port 2 in the even-mode circuit. Since the even-mode circuit is lossless and reciprocal we have

$$T_e = T_o \qquad (T \text{ is the transmission coefficient}) \tag{1a}$$

$$|\Gamma_e| = |\Gamma_o|. \tag{1b}$$

The response of the four-port to a unit wave in port 1 is given in Fig. 2, expressed in the cascade parameters of the even-mode circuit (1-$\Omega$ terminations are used).

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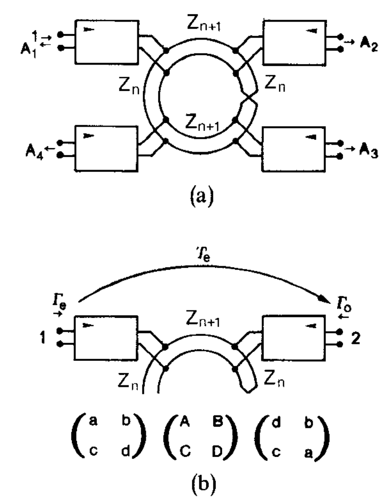


$$\begin{pmatrix} A_e & B_e \\ C_e & D_e \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} d & b \\ c & a \end{pmatrix}$$

$$A_1 = \frac{\Gamma_e + \Gamma_o}{2} = \frac{B_e - C_e}{A_e + B_e + C_e + D_e}$$

$$A_2 = \frac{T_e + T_o}{2} = \frac{2}{A_e + B_e + C_e + D_e}$$

$$A_3 = \frac{T_e - T_o}{2} = 0$$

$$A_4 = \frac{\Gamma_e - \Gamma_o}{2} = \frac{A_e - D_e}{A_e + B_e + C_e + D_e}$$

$$= \frac{A - D}{A_e + B_e + C_e + D_e} = \frac{2Z_{n+1}/Z_n}{A_e + B_e + C_e + D_e}$$

Fig. 2. The response to a unit wave of a reversed-phase hybrid ring with four equal two-ports connected. (a) Total circuit. (b) Even-mode circuit.

As for the basic hybrid ring the isolation is perfect for all frequencies ($A_3 = 0$). For reciprocal two-ports the numerator of $A_4$ ($= A_e - D_e = A - D$) is equal to a constant. Thus the power division ratio is constant, independent of frequency. The phase difference between output signals is also frequency independent. If $A_1$ were zero, the match would be perfect and we would have an ideal directional coupler. $A_1$ is not zero, but the two-ports can be designed to improve the match of the basic hybrid. With the notations of Fig. 2 we have:

$$\text{coupling to port } 4 = C \tag{2a}$$

(at frequencies of perfect match)

$$\text{power division ratio} = (A_4/A_2)^2 = R^2 \tag{2b}$$

$$R = \frac{Z_{n+1}}{Z_n} \tag{2c}$$

$$C = 10 \log \left( \frac{1}{R^2} + 1 \right). \tag{2d}$$

Since the even-mode two-port is lossless

$$|\Gamma_e|^2 + |T_e|^2 = 1. \tag{2e}$$

Since the four-port is lossless

$$|A_1|^2 + |A_2|^2 + |A_4|^2 = 1 \tag{2f}$$

or

$$|A_1|^2 + |A_2|^2(1 + R^2) = 1. \tag{2g}$$

Equation (2g) shows that:

1) every frequency with perfect match has perfect coupling,

2) if the VSWR varies with equal ripples, then the coupling will have equal ripples too.

### B. The Basic Reversed-Phase Hybrid Ring

Unlike the "rat race" an equal ripple performance can be obtained by decreasing the impedance level of the basic hybrid ring. With the notations of Fig. 2 we have

$$Z_2 = \sqrt{1 + R^2} \tag{3a}$$

which gives a maximally flat VSWR response,

$$Z_2 < \sqrt{1 + R^2} \tag{3b}$$

which gives an equal ripple VSWR response,

$$\text{VSWR} = \frac{1 + R^2}{Z_2^2}, \tag{3c}$$

at the design frequency when $Z_2 \leq \sqrt{1 + R^2}$. Equal power division ($C = 3.01$ dB) is obtained when $Z_1 = Z_2$. $Z_1 = Z_2 = \sqrt{2}\ \Omega$ yields a maximally flat VSWR performance (1-Ω terminations).

Theoretical curves of this example and some other 3-dB reversed-phase hybrid rings are shown in Fig. 3.

### C. Stubs for Match Improvement of a Reversed-Phase Hybrid Ring

The method of match improvement by a cascade of short-circuited shunt stubs and open-circuited series stubs was reported on by Podell [6]. Fig. 4 shows how the stubs are connected. The length of each stub is a quarter-wave at the design frequency. Four equal circuits are used as described in Section II-A with port 2 connected to the basic hybrid.

The stubs have no influence at the center frequency, where we have

$$\text{VSWR} = \frac{1 + R^2}{Z_{n+1}^2} \tag{4a}$$

at the design frequency when $Z_{n+1} \leq \sqrt{1 + R^2}$ and

$$\text{VSWR} = \frac{Z_{n+1}^2}{1 + R^2} \tag{4b}$$

at the design frequency when $Z_{n+1} \geq \sqrt{1 + R^2}$; $R = Z_{n+1}/Z_n$ as defined in Section II-A.

The bandwidth is increased considerably when the appropriate impedances of the stubs are found. Fig. 5 shows the performance of a 3-dB coupler with $n = 4$, that is when three stubs are used on each port.

No synthesis procedure was given in [6], and here the stub impedances have been calculated by computer optimiza-

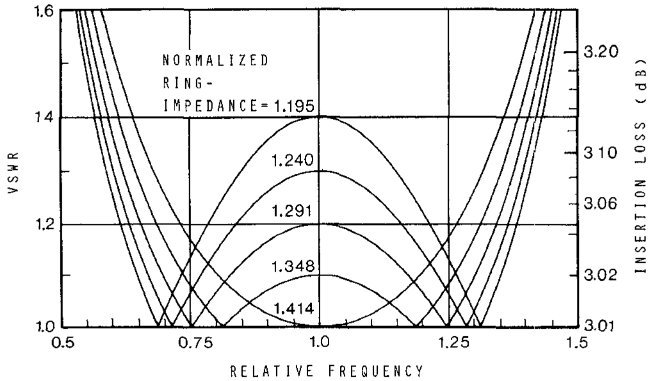


Fig. 3. Theoretical curves of 3-dB reversed-phase hybrid rings.

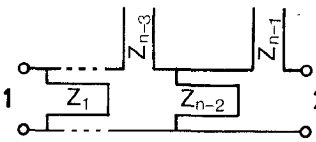


Fig. 4. Stub network consisting of cascaded short-circuited shunt stubs and open-circuited series stubs.

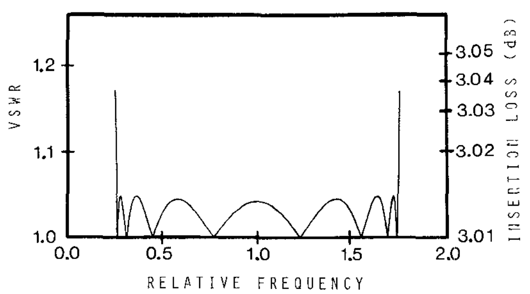


Fig. 5. Theoretical curves of a 3-dB hybrid ring with stubs for matching: $n = 4$, $Z_1 = 0.280\ \Omega$, $Z_2 = 1.778\ \Omega$, $Z_3 = 0.692\ \Omega$, $Z_4 = Z_5 = 1.444\ \Omega$.

tion. The bandwidth is 6.7 : 1 for VSWR $\leq 1.05$ and a coupling ripple of $+0.005$ dB. These figures are to be compared with those of a CTL coupler and a BL coupler given in the Introduction.

### D. Stubs for Match Improvement of a BL Coupler

Match improvement by stubs has been tried for other couplers as well. It was found that a great improvement is possible for a BL coupler. Four equal open-circuited series stubs, one at each port of a coupler with two branches, give a significant improvement in bandwidth. With branch admittances $Y_1$ and $Y_2$ we find that maximally flat performance requires

$$Y_2^2 - Y_1^2 = 1 \tag{5a}$$

$$Z_{\text{stub}} = Y_1 + Y_2 \tag{5b}$$

where $Z_{\text{stub}}$ is the stub impedance. The terminations are $1\ \Omega$.

### E. Cascaded Transmission Lines for Match Improvement of a Reversed-Phase Hybrid Ring

A new circuit for match improvement of a reversed-phase hybrid ring is presented. A cascade of transmission lines is connected to each port of a reversed-phase hybrid ring as

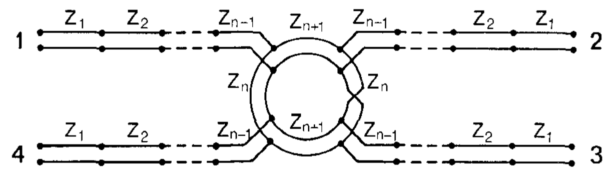


Fig. 6. A wide-band hybrid consisting of a reversed-phase hybrid ring and transmission-line cascades.

Fig. 6 shows. The length of each line is a quarter-wave at the design frequency.

This device has been studied in detail and a complete synthesis procedure for Butterworth and Chebyshev performance has been developed. The analysis is made with even and odd modes. Some general results of such an analysis have already been given in Section II-A. The even-mode circuit of the device in Fig. 6 consists of two stubs and $2n - 1$ transmission lines, each having the electric length $2\theta$; $\theta = 45^\circ$ at the design frequency. The open-circuited and short-circuited stubs each have the electric length $\theta$.

We define the Richards' variable $s \doteq j \tan \theta$. By analyzing the even-mode circuit and using induction, the cascade matrix is found to have the following form:

$$\begin{pmatrix} A_e & B_e \\ C_e & D_e \end{pmatrix} = \frac{1}{(1 - s^2)^{2n-1}} \begin{pmatrix} A_{2n-1}(s^2) & sB_{2n-2}(s^2) \\ \dfrac{1}{s} C_{2n}(s^2) & D_{2n-1}(s^2) \end{pmatrix}. \tag{6}$$

The subscripts give the degree of the polynomials $A(s^2)$, $B(s^2)$, $C(s^2)$, and $D(s^2)$.

We also find

$$\frac{\Gamma_e}{T_e} = \frac{A_e + B_e - (C_e + D_e)}{2}$$

$$= \frac{A_4}{A_2} + \frac{A_1}{A_2} = R - k \frac{P_{2n}(\cos 2\theta)}{j \sin 2\theta} \tag{7}$$

where $P_{2n}$ is given in [3] and [11].

For Butterworth performance we specify

$$P_{2n}(x) = x^{2n} \tag{8a}$$

$$x = \cos 2\theta \tag{8b}$$

$$k = A_k, \qquad \text{a positive constant.} \tag{8c}$$

For Chebyshev performance we specify

$$P_{2n}(x) = \tfrac{1}{2}[1 + \sqrt{1 - x_c^2}]T_{2n}\left(\frac{x}{x_c}\right) - \tfrac{1}{2}[1 - \sqrt{1 - x_c^2}]T_{2n-2}\left(\frac{x}{x_c}\right) \tag{9a}$$

$$x = \cos 2\theta \tag{9b}$$

$$x_c = \cos 2\theta_c = \cos (f_c \cdot 90^\circ) \tag{9c}$$

$$k = a_k, \qquad \text{a positive constant} \tag{9d}$$

where $f_c$ is the upper band edge frequency normalized to the center frequency, and $T_n$ is the Chebyshev polynomial of the first kind of degree $n$. In the band $-x_c \le x \le x_c$ $(P_{2n}(x))/(\sqrt{1-x^2})$ has $2n$ zeros and $2n-1$ equal ripples. By observing that $(P_{2n}(x_c))/(\sqrt{1-x_c^2}) = 1$, $a_k$ has been calculated and is expressed in terms of $R$ and the VSWR of the hybrid:

$$a_k = \sqrt{\frac{1+R^2}{\left(\frac{\text{VSWR}+1}{\text{VSWR}-1}\right)^2 - 1}} \tag{10}$$

where VSWR is the maximum VSWR of the hybrid in the equal ripple frequency band, and $R$ is defined in (2b).

Expressing $\cos 2\theta$ and $\sin 2\theta$ in the Richards' variable $s = j \tan \theta$ gives

$$\cos 2\theta = \frac{1+s^2}{1-s^2} \tag{11a}$$

$$j \sin 2\theta = \frac{2s}{1-s^2}. \tag{11b}$$

The usual way to determine $k$ and $x_c$ is to calculate $\Gamma_e/T_e$ at zero frequency, but this cannot be used here, since there is a short-circuited shunt stub in the even-mode circuit, resulting in $\Gamma_e = -1$ and $T_e = 0$. $k$ and $x_c$ can be expressed in terms of the impedance of the shunt stub $= Z_n$, but $Z_n$ is not known until the synthesis is completed. We find in the Butterworth case

$$A_k = \frac{1}{Z_n} \tag{12}$$

and in the Chebyshev case

$$a_k \cdot P_{2n}(1) = \frac{1}{Z_n}. \tag{13}$$

We also know that $R = Z_{n+1}/Z_n$.

To specify $\Gamma_e/T_e$ we guess the initial value of $A_k$ in the Butterworth case and $f_c$ (or $a_k$) in the Chebyshev case. Once $\Gamma_e/T_e$ is specified, the synthesis procedure is straightforward. The modulus squared of the even-mode reflection coefficient is determined by

$$|\Gamma_e|^2 = \frac{\left|\frac{\Gamma_e}{T_e}\right|^2}{1 + \left|\frac{\Gamma_e}{T_e}\right|^2} \tag{14}$$

since $|\Gamma_e|^2 + |T_e|^2 = 1$.

The zeros of $\Gamma_e$ are given by the zeros of $\Gamma_e/T_e$. The numerator of $\Gamma_e$ is given directly by the numerator of $\Gamma_e/T_e$ and it is not necessary to calculate the zeros explicitly.

The poles of $\Gamma_e$ are given by the zeros of $1 + |\Gamma_e/T_e|^2$ that lie in the left half of the $s$ plane. Computer calculations give the even-mode reflection coefficient $\Gamma_e$. The input impedance $Z_e$ of the even-mode network is then given by

$$Z_e = \frac{1+\Gamma_e}{1-\Gamma_e} = \frac{A_e + B_e}{C_e + D_e}. \tag{15}$$

The cascade matrix parameters $A_e$, $B_e$, $C_e$, and $D_e$ are then determined by using the fact that $A_e$ and $D_e$ are even polynomials, but $B_e$ and $C_e$ are odd polynomials in $s$ [(6)], and the equation

$$A_e D_e - B_e C_e = 1. \tag{16}$$

The synthesis starts by extracting a double length $(2\theta)$ of transmission line from each side of the even-mode circuit.

The condition for extracting a transmission line with characteristic impedance $Z_k$ of double length $(2\theta)$ from the even-mode circuit is

$$\left.\frac{A_{2n-1}(s^2)}{C_{2n}(s^2)}\right|_{s^2=1} = Z_k \tag{17}$$

$$\left.\frac{B_{2n-2}(s^2)}{D_{2n-1}(s^2)}\right|_{s^2=1} = Z_k \tag{18}$$

$$\left(\frac{d}{ds}\frac{sA_{2n-1}}{C_{2n}}\right)_{s^2=1} = 0 \tag{19}$$

$$\left(\frac{d}{ds}\frac{sB_{2n-2}}{D_{2n-1}}\right)_{s^2=1} = 0. \tag{20}$$

The condition for extracting a line from the other side of the even-mode circuit is given by switching $A$ and $D$ in (17)–(20). This procedure is then repeated until $n-1$ lines have been extracted from each side. The remaining circuit is then an even-mode circuit similar to that of the basic hybrid. If the correct values of $A_k$ (Butterworth case) or $f_c$ (Chebyshev case) have been chosen, $Z_n$ and $Z_n + 1$ are easily found. If $A_k$ (or $f_c$) differ from the correct values, there will also be a transformer with ratio $N$ dividing the $Z_n + 1$ transmission line into two equal parts. An ordinary zero searching routine is then used to find the correct values of $A_k$ (or $f_c$) resulting in the transformer ratio $N = 1$. That is, vary $A_k$ (or $f_c$) and repeat the synthesis procedure until the transformer is eliminated. Expressed in equation form we have the following. For the Butterworth case find $A_k$ so that

$$f(A_k) = N(A_k) - 1 = 0. \tag{21}$$

For the Chebyshev case find $f_c$ so that

$$f(f_c) = N(f_c) - 1 = 0. \tag{22}$$

Alternatively, $f_c$ can be specified and $a_k$ varied; therefore, find $a_k$ so that

$$f(a_k) = N(a_k) - 1 = 0. \tag{23}$$

$N \neq 1$ can be used in practice, when different load impedances are used in the hybrid.

### *F. Tables and Curves of the New Device*

Tables I and II give characteristic impedances calculated by the synthesis procedure described in Section II-E. Fig. 7 shows the theoretical performance of some 3-dB hybrids of the type shown in Fig. 6. The corresponding parameters are found in Table I and Table II(a).

## III. Practical Considerations

### *A. Realization of the Basic Hybrid Ring*

The difficulty with this device is the practical realization. The best agreement with theory is obtained by using twin

TABLE I
NORMALIZED CHARACTERISTIC IMPEDANCES OF HYBRIDS AS SHOWN IN FIG. 6—BUTTERWORTH CASE

| $Z_1 - Z_{n-1}$ (ohms) | | | | | $Z_{n+1}$ (ohms) | $Z_n$ (ohms) | Coupling (dB) | $A_k$ | n |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1.4142 | 1.4142 | 3.01 | 0.7071 | 1 |
| | | | | 0.9020 | 1.1506 | 1.1506 | 3.01 | 0.8691 | 2 |
| | | | 0.9706 | 0.8050 | 0.9730 | 0.9730 | 3.01 | 1.0278 | 3 |
| | | 0.9915 | 0.9246 | 0.7180 | 0.8385 | 0.8385 | 3.01 | 1.1926 | 4 |
| | 0.9976 | 0.9735 | 0.8704 | 0.6412 | 0.7310 | 0.7310 | 3.01 | 1.3681 | 5 |
| 0.9993 | 0.9912 | 0.9471 | 0.8129 | 0.5736 | 0.6421 | 0.6421 | 3.01 | 1.5575 | 6 |
| | | | | | 1.1547 | 2.0000 | 6.02 | 0.5000 | 1 |
| | | | | 0.9128 | 0.9621 | 1.6665 | 6.02 | 0.6001 | 2 |
| | | | 0.9743 | 0.8264 | 0.8306 | 1.4387 | 6.02 | 0.6951 | 3 |
| | | 0.9927 | 0.9344 | 0.7483 | 0.7297 | 1.2639 | 6.02 | 0.7912 | 4 |
| | 0.9980 | 0.9774 | 0.8874 | 0.6787 | 0.6480 | 1.1224 | 6.02 | 0.8910 | 5 |
| 0.9994 | 0.9926 | 0.9550 | 0.8373 | 0.6169 | 0.5796 | 1.0038 | 6.02 | 0.9962 | 6 |
| | | | | | 1.0541 | 3.1623 | 10.00 | 0.3162 | 1 |
| | | | | 0.9360 | 0.9235 | 2.7706 | 10.00 | 0.3609 | 2 |
| | | | 0.9820 | 0.8723 | 0.8318 | 2.4955 | 10.00 | 0.4007 | 3 |
| | | 0.9951 | 0.9542 | 0.8141 | 0.7597 | 2.2791 | 10.00 | 0.4388 | 4 |
| | 0.9987 | 0.9848 | 0.9215 | 0.7613 | 0.6998 | 2.0993 | 10.00 | 0.4764 | 5 |
| 0.9996 | 0.9952 | 0.9700 | 0.8865 | 0.7136 | 0.6484 | 1.9452 | 10.00 | 0.5141 | 6 |
| | | | | | 1.0162 | 5.6234 | 15.00 | 0.1778 | 1 |
| | | | | 0.9604 | 0.9372 | 5.1864 | 15.00 | 0.1928 | 2 |
| | | | 0.9893 | 0.9208 | 0.8803 | 4.8712 | 15.00 | 0.2053 | 3 |
| | | 0.9972 | 0.9730 | 0.8842 | 0.8344 | 4.6172 | 15.00 | 0.2166 | 4 |
| | 0.9993 | 0.9914 | 0.9539 | 0.8506 | 0.7954 | 4.4014 | 15.00 | 0.2272 | 5 |
| 0.9998 | 0.9974 | 0.9832 | 0.9334 | 0.8197 | 0.7612 | 4.2125 | 15.00 | 0.2374 | 6 |

line or balanced line [7], [8]. The perfect180° phase shift is then obtained by mechanically twisting the two lines 180°.

Another approach to realize the 180° phase shift is to use a shorted parallel-coupled line section [12]. The equivalent circuit of this device is the same as the one shown in Fig. 1 except that additional quarter-wave shorted stubs are parallel-coupled to ports 2 and 3 [13]. Finite values of the stub impedances (caused by finite coupling in the coupled section) result in a degradation of performance.

Another realization utilizes a combination of microstrip and microslot transmission lines [14].

## B. Realization of Stubs

The realization of open-circuited series stubs in stripline or microstrip is not very easy. One method is to place the

TABLE II
NORMALIZED CHARACTERISTIC IMPEDANCES OF HYBRIDS AS SHOWN IN FIG. 6—CHEBYSHEV CASE.
(a) COUPLING 3 dB. (b) COUPLING 6 dB. (c) COUPLING 10 dB. (d) COUPLING 15 dB.

| $Z_1 - Z_{n-1}$ (ohms) | | | | $Z_{n+1}$ (ohms) | $Z_n$ (ohms) | Coupling (dB) | VSWR | Relative bandwidth | $f_c$ | n |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1.407 | 1.407 | 3.01 | 1.01 | 1.20 | 1.089 | 1 |
| | | | 0.871 | 1.078 | 1.078 | 3.01 | 1.01 | 1.88 | 1.305 | 2 |
| | | 0.926 | 0.701 | 0.805 | 0.805 | 3.01 | 1.01 | 2.62 | 1.448 | 3 |
| | 0.949 | 0.787 | 0.532 | 0.585 | 0.585 | 3.01 | 1.01 | 3.32 | 1.537 | 4 |
| 0.960 | 0.836 | 0.622 | 0.387 | 0.413 | 0.413 | 3.01 | 1.01 | 3.95 | 1.596 | 5 |
| | | | | 1.380 | 1.380 | 3.01 | 1.05 | 1.48 | 1.193 | 1 |
| | | | 0.828 | 0.993 | 0.993 | 3.01 | 1.05 | 2.50 | 1.429 | 2 |
| | | 0.876 | 0.621 | 0.694 | 0.694 | 3.01 | 1.05 | 3.43 | 1.549 | 3 |
| | 0.898 | 0.691 | 0.439 | 0.472 | 0.472 | 3.01 | 1.05 | 4.22 | 1.617 | 4 |
| 0.910 | 0.734 | 0.504 | 0.297 | 0.312 | 0.312 | 3.01 | 1.05 | 4.88 | 1.660 | 5 |
| | | | | 1.348 | 1.348 | 3.01 | 1.10 | 1.71 | 1.262 | 1 |
| | | | 0.793 | 0.932 | 0.932 | 3.01 | 1.10 | 2.90 | 1.488 | 2 |
| | | 0.836 | 0.571 | 0.628 | 0.628 | 3.01 | 1.10 | 3.90 | 1.592 | 3 |
| | 0.857 | 0.631 | 0.388 | 0.413 | 0.413 | 3.01 | 1.10 | 4.70 | 1.649 | 4 |
| 0.870 | 0.669 | 0.441 | 0.253 | 0.263 | 0.263 | 3.01 | 1.10 | 5.35 | 1.685 | 5 |
| | | | | 1.291 | 1.291 | 3.01 | 1.20 | 2.05 | 1.344 | 1 |
| | | | 0.742 | 0.852 | 0.852 | 3.01 | 1.20 | 3.40 | 1.546 | 2 |
| | | 0.779 | 0.509 | 0.552 | 0.552 | 3.01 | 1.20 | 4.43 | 1.632 | 3 |
| | 0.798 | 0.558 | 0.332 | 0.349 | 0.349 | 3.01 | 1.20 | 5.22 | 1.679 | 4 |
| 0.809 | 0.589 | 0.371 | 0.208 | 0.215 | 0.215 | 3.01 | 1.20 | 5.84 | 1.707 | 5 |
| | | | | 1.240 | 1.240 | 3.01 | 1.30 | 2.31 | 1.396 | 1 |
| | | | 0.702 | 0.796 | 0.796 | 3.01 | 1.30 | 3.73 | 1.577 | 2 |
| | | 0.735 | 0.468 | 0.503 | 0.503 | 3.01 | 1.30 | 4.77 | 1.653 | 3 |
| | 0.753 | 0.510 | 0.298 | 0.311 | 0.311 | 3.01 | 1.30 | 5.54 | 1.694 | 4 |
| | | | | 1.195 | 1.195 | 3.01 | 1.40 | 2.53 | 1.433 | 1 |
| | | | 0.670 | 0.751 | 0.751 | 3.01 | 1.40 | 3.98 | 1.598 | 2 |
| | | 0.700 | 0.437 | 0.467 | 0.467 | 3.01 | 1.40 | 5.01 | 1.667 | 3 |
| | 0.716 | 0.474 | 0.273 | 0.284 | 0.284 | 3.01 | 1.40 | 5.75 | 1.704 | 4 |
| | | | | 1.155 | 1.155 | 3.01 | 1.50 | 2.70 | 1.460 | 1 |
| | | | 0.642 | 0.715 | 0.715 | 3.01 | 1.50 | 4.18 | 1.614 | 2 |
| | | 0.670 | 0.413 | 0.438 | 0.438 | 3.01 | 1.50 | 5.19 | 1.677 | 3 |
| | 0.685 | 0.446 | 0.254 | 0.263 | 0.263 | 3.01 | 1.50 | 5.92 | 1.711 | 4 |

(a)

| $Z_1 - Z_{n-1}$ (ohms) | | | | $Z_{n+1}$ (ohms) | $Z_n$ (ohms) | Coupling (dB) | VSWR | Relative bandwidth | $f_c$ | n |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1.149 | 1.990 | 6.02 | 1.01 | 1.21 | 1.096 | 1 |
| | | | 0.883 | 0.905 | 1.567 | 6.02 | 1.01 | 1.93 | 1.318 | 2 |
| | | 0.933 | 0.727 | 0.698 | 1.209 | 6.02 | 1.01 | 2.71 | 1.462 | 3 |
| | 0.953 | 0.808 | 0.571 | 0.526 | 0.912 | 6.02 | 1.01 | 3.45 | 1.551 | 4 |
| 0.963 | 0.852 | 0.658 | 0.432 | 0.388 | 0.673 | 6.02 | 1.01 | 4.12 | 1.609 | 5 |
| | | | | 1.127 | 1.952 | 6.02 | 1.05 | 1.52 | 1.206 | 1 |
| | | | 0.841 | 0.837 | 1.449 | 6.02 | 1.05 | 2.61 | 1.445 | 2 |
| | | 0.885 | 0.650 | 0.608 | 1.053 | 6.02 | 1.05 | 3.59 | 1.565 | 3 |
| | 0.905 | 0.716 | 0.478 | 0.432 | 0.748 | 6.02 | 1.05 | 4.44 | 1.632 | 4 |
| 0.917 | 0.757 | 0.543 | 0.339 | 0.300 | 0.520 | 6.02 | 1.05 | 5.16 | 1.675 | 5 |
| | | | | 1.101 | 1.907 | 6.02 | 1.10 | 1.78 | 1.280 | 1 |
| | | | 0.807 | 0.788 | 1.365 | 6.02 | 1.10 | 3.05 | 1.506 | 2 |
| | | 0.846 | 0.600 | 0.554 | 0.959 | 6.02 | 1.10 | 4.11 | 1.609 | 3 |
| | 0.866 | 0.658 | 0.427 | 0.382 | 0.661 | 6.02 | 1.10 | 4.98 | 1.666 | 4 |
| 0.877 | 0.694 | 0.480 | 0.293 | 0.257 | 0.445 | 6.02 | 1.10 | 5.69 | 1.701 | 5 |
| | | | | 1.054 | 1.826 | 6.02 | 1.20 | 2.16 | 1.367 | 1 |
| | | | 0.756 | 0.723 | 1.253 | 6.02 | 1.20 | 3.60 | 1.565 | 2 |
| | | 0.790 | 0.539 | 0.490 | 0.849 | 6.02 | 1.20 | 4.71 | 1.650 | 3 |
| | 0.808 | 0.586 | 0.369 | 0.326 | 0.565 | 6.02 | 1.20 | 5.58 | 1.696 | 4 |
| 0.819 | 0.617 | 0.409 | 0.244 | 0.213 | 0.368 | 6.02 | 1.20 | 6.26 | 1.725 | 5 |
| | | | | 1.013 | 1.754 | 6.02 | 1.30 | 2.45 | 1.421 | 1 |
| | | | 0.717 | 0.677 | 1.172 | 6.02 | 1.30 | 3.97 | 1.598 | 2 |
| | | 0.747 | 0.498 | 0.449 | 0.777 | 6.02 | 1.30 | 5.09 | 1.671 | 3 |
| | 0.764 | 0.538 | 0.332 | 0.293 | 0.507 | 6.02 | 1.30 | 5.94 | 1.712 | 4 |
| | | | | 0.976 | 1.690 | 6.02 | 1.40 | 2.69 | 1.458 | 1 |
| | | | 0.685 | 0.640 | 1.109 | 6.02 | 1.40 | 4.25 | 1.619 | 2 |
| | | 0.712 | 0.466 | 0.418 | 0.724 | 6.02 | 1.40 | 5.36 | 1.686 | 3 |
| | 0.727 | 0.502 | 0.306 | 0.269 | 0.465 | 6.02 | 1.40 | 6.19 | 1.722 | 4 |
| | | | | 0.943 | 1.633 | 6.02 | 1.50 | 2.89 | 1.486 | 1 |
| | | | 0.657 | 0.610 | 1.057 | 6.02 | 1.50 | 4.47 | 1.634 | 2 |
| | | 0.682 | 0.441 | 0.394 | 0.682 | 6.02 | 1.50 | 5.57 | 1.696 | 3 |
| | 0.696 | 0.473 | 0.286 | 0.250 | 0.433 | 6.02 | 1.50 | 6.38 | 1.729 | 4 |

(b)

| $Z_1 - Z_{n-1}$ (ohms) | | | | $Z_{n+1}$ (ohms) | $Z_n$ (ohms) | Coupling (dB) | VSWR | Relative bandwidth | $f_c$ | n |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1.049 | 3.146 | 10.00 | 1.01 | 1.26 | 1.115 | 1 |
| | | | 0.909 | 0.876 | 2.628 | 10.00 | 1.01 | 2.08 | 1.350 | 2 |
| | | 0.947 | 0.786 | 0.723 | 2.168 | 10.00 | 1.01 | 2.96 | 1.495 | 3 |
| | 0.963 | 0.851 | 0.659 | 0.589 | 1.767 | 10.00 | 1.01 | 3.80 | 1.583 | 4 |
| 0.971 | 0.887 | 0.735 | 0.541 | 0.475 | 1.424 | 10.00 | 1.01 | 4.58 | 1.642 | 5 |
| | | | | 1.029 | 3.086 | 10.00 | 1.05 | 1.65 | 1.246 | 1 |
| | | | 0.871 | 0.819 | 2.456 | 10.00 | 1.05 | 2.91 | 1.488 | 2 |
| | | 0.904 | 0.716 | 0.645 | 1.934 | 10.00 | 1.05 | 4.05 | 1.604 | 3 |
| | 0.920 | 0.772 | 0.572 | 0.503 | 1.508 | 10.00 | 1.05 | 5.05 | 1.669 | 4 |
| 0.930 | 0.805 | 0.632 | 0.448 | 0.388 | 1.163 | 10.00 | 1.05 | 5.93 | 1.711 | 5 |
| | | | | 1.005 | 3.015 | 10.00 | 1.10 | 1.99 | 1.330 | 1 |
| | | | 0.838 | 0.776 | 2.329 | 10.00 | 1.10 | 3.47 | 1.552 | 2 |
| | | 0.869 | 0.669 | 0.596 | 1.789 | 10.00 | 1.10 | 4.71 | 1.650 | 3 |
| | 0.884 | 0.718 | 0.521 | 0.454 | 1.362 | 10.00 | 1.10 | 5.77 | 1.704 | 4 |
| 0.894 | 0.749 | 0.571 | 0.397 | 0.342 | 1.026 | 10.00 | 1.10 | 6.66 | 1.739 | 5 |
| | | | | 0.962 | 2.887 | 10.00 | 1.20 | 2.50 | 1.428 | 1 |
| | | | 0.789 | 0.719 | 2.156 | 10.00 | 1.20 | 4.19 | 1.614 | 2 |
| | | 0.816 | 0.609 | 0.537 | 1.610 | 10.00 | 1.20 | 5.51 | 1.693 | 3 |
| | 0.830 | 0.651 | 0.460 | 0.398 | 1.194 | 10.00 | 1.20 | 6.59 | 1.736 | 4 |
| 0.839 | 0.677 | 0.501 | 0.342 | 0.293 | 0.878 | 10.00 | 1.20 | 7.47 | 1.764 | 5 |
| | | | | 0.925 | 2.773 | 10.00 | 1.30 | 2.89 | 1.486 | 1 |
| | | | 0.750 | 0.677 | 2.030 | 10.00 | 1.30 | 4.68 | 1.648 | 2 |
| | | 0.774 | 0.567 | 0.497 | 1.490 | 10.00 | 1.30 | 6.03 | 1.716 | 3 |
| | 0.787 | 0.604 | 0.421 | 0.363 | 1.088 | 10.00 | 1.30 | 7.10 | 1.753 | 4 |
| | | | | 0.891 | 2.672 | 10.00 | 1.40 | 3.22 | 1.526 | 1 |
| | | | 0.718 | 0.643 | 1.929 | 10.00 | 1.40 | 5.06 | 1.670 | 2 |
| | | 0.740 | 0.535 | 0.466 | 1.399 | 10.00 | 1.40 | 6.41 | 1.730 | 3 |
| | 0.752 | 0.567 | 0.392 | 0.337 | 1.010 | 10.00 | 1.40 | 7.47 | 1.764 | 4 |
| | | | | 0.861 | 2.582 | 10.00 | 1.50 | 3.50 | 1.555 | 1 |
| | | | 0.690 | 0.615 | 1.844 | 10.00 | 1.50 | 5.37 | 1.686 | 2 |
| | | 0.710 | 0.509 | 0.442 | 1.325 | 10.00 | 1.50 | 6.71 | 1.741 | 3 |
| | 0.721 | 0.538 | 0.369 | 0.316 | 0.948 | 10.00 | 1.50 | 7.75 | 1.771 | 4 |

(c)

| $Z_1 - Z_{n-1}$ (ohms) | | | | $Z_{n+1}$ (ohms) | $Z_n$ (ohms) | Coupling (dB) | VSWR | Relative bandwidth | $f_c$ | n |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1.011 | 5.593 | 15.00 | 1.01 | 1.35 | 1.150 | 1 |
| | | | 0.938 | 0.899 | 4.976 | 15.00 | 1.01 | 2.34 | 1.401 | 2 |
| | | 0.963 | 0.854 | 0.795 | 4.400 | 15.00 | 1.01 | 3.38 | 1.543 | 3 |
| | 0.973 | 0.898 | 0.764 | 0.700 | 3.873 | 15.00 | 1.01 | 4.38 | 1.628 | 4 |
| 0.978 | 0.922 | 0.819 | 0.677 | 0.614 | 3.396 | 15.00 | 1.01 | 5.31 | 1.683 | 5 |
| | | | | 0.992 | 5.487 | 15.00 | 1.05 | 1.92 | 1.315 | 1 |
| | | | 0.904 | 0.851 | 4.708 | 15.00 | 1.05 | 3.46 | 1.552 | 2 |
| | | 0.926 | 0.794 | 0.729 | 4.034 | 15.00 | 1.05 | 4.85 | 1.658 | 3 |
| | 0.937 | 0.833 | 0.688 | 0.624 | 3.452 | 15.00 | 1.05 | 6.10 | 1.718 | 4 |
| 0.943 | 0.857 | 0.735 | 0.592 | 0.532 | 2.945 | 15.00 | 1.05 | 7.22 | 1.757 | 5 |
| | | | | 0.969 | 5.361 | 15.00 | 1.10 | 2.43 | 1.416 | 1 |
| | | | 0.874 | 0.814 | 4.505 | 15.00 | 1.10 | 4.26 | 1.620 | 2 |
| | | 0.894 | 0.752 | 0.686 | 3.795 | 15.00 | 1.10 | 5.81 | 1.706 | 3 |
| | 0.905 | 0.788 | 0.641 | 0.578 | 3.197 | 15.00 | 1.10 | 7.15 | 1.755 | 4 |
| 0.911 | 0.810 | 0.682 | 0.543 | 0.486 | 2.687 | 15.00 | 1.10 | 8.33 | 1.786 | 5 |
| | | | | 0.928 | 5.133 | 15.00 | 1.20 | 3.22 | 1.526 | 1 |
| | | | 0.827 | 0.762 | 4.213 | 15.00 | 1.20 | 5.33 | 1.684 | 2 |
| | | 0.845 | 0.696 | 0.630 | 3.485 | 15.00 | 1.20 | 7.01 | 1.750 | 3 |
| | 0.854 | 0.726 | 0.582 | 0.521 | 2.884 | 15.00 | 1.20 | 8.42 | 1.788 | 4 |
| 0.861 | 0.746 | 0.615 | 0.484 | 0.431 | 2.385 | 15.00 | 1.20 | 9.64 | 1.812 | 5 |
| | | | | 0.891 | 4.931 | 15.00 | 1.30 | 3.86 | 1.588 | 1 |
| | | | 0.789 | 0.722 | 3.995 | 15.00 | 1.30 | 6.10 | 1.718 | 2 |
| | | 0.805 | 0.655 | 0.590 | 3.266 | 15.00 | 1.30 | 7.83 | 1.773 | 3 |
| | 0.814 | 0.682 | 0.541 | 0.484 | 2.676 | 15.00 | 1.30 | 9.25 | 1.805 | 4 |
| | | | | 0.859 | 4.752 | 15.00 | 1.40 | 4.39 | 1.629 | 1 |
| | | | 0.757 | 0.689 | 3.815 | 15.00 | 1.40 | 6.69 | 1.740 | 2 |
| | | 0.771 | 0.622 | 0.559 | 3.094 | 15.00 | 1.40 | 8.44 | 1.788 | 3 |
| | 0.780 | 0.646 | 0.510 | 0.455 | 2.516 | 15.00 | 1.40 | 9.87 | 1.816 | 4 |
| | | | | 0.830 | 4.591 | 15.00 | 1.50 | 4.84 | 1.658 | 1 |
| | | | 0.729 | 0.662 | 3.661 | 15.00 | 1.50 | 7.18 | 1.756 | 2 |
| | | 0.742 | 0.595 | 0.533 | 2.952 | 15.00 | 1.50 | 8.94 | 1.799 | 3 |
| | 0.750 | 0.617 | 0.484 | 0.431 | 2.387 | 15.00 | 1.50 | 10.35 | 1.824 | 4 |

(d)

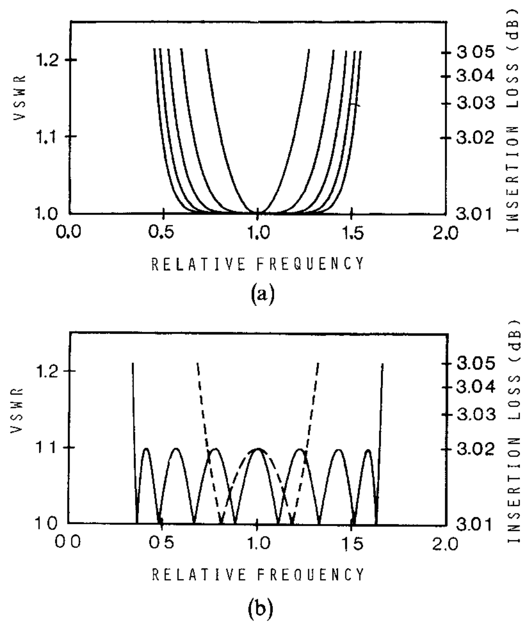


Fig. 7. Theoretical performance of 3-dB hybrids as shown in Fig. 6. (a) Butterworth case $n = 1, 2, 3, 4, 5$. (b) Chebyshev case $n = 1, 4$.

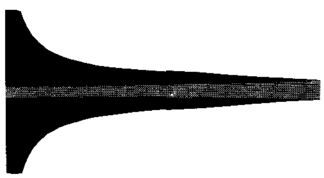

Fig. 8. Transition from a microstrip to a balanced line using two quarter-wave transmission lines.

stubs in the ground plane. Another possibility is to use coupled lines. Several coupled-line structures are shown in [13], [15], and [16]. For the complete unit the problem of realizing the perfect 180° phase shift in the hybrid still remains.

### C. Realization of the New Device

The use of transmission-line cascades to improve the bandwidth of the basic hybrid ring also makes the realization easier. The transmission-line cascades are used to make a transition between balanced line and microstrip as Fig. 8 shows. The basic hybrid ring is then made in accordance with [7] and then connected to the four equal transitions/transmission-line cascades.

## IV. Conclusions

A new wide-band coupler has been presented. The device is a development of the reversed-phase hybrid ring. The realization of 3-dB couplers with wide-band performance is possible with the new device. Planar transmission-line techniques can be used in manufacturing the coupler and the only critical part is the 180° phase change in one branch of the basic hybrid. The new device may be realized for microwave frequencies [7] and even millimeter-wave realizations are possible [8]. The new device does not require very tight coupling as does the CTL coupler, nor does it require very high and very low impedance levels simultaneously as does the BL coupler. The frequency-independent properties of the device makes it advantageous for wide-band applications.

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## References

[1] P. P. Toulios and A. C. Todd, "Synthesis of symmetrical TEM-mode directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 536–544, Sept. 1965.
[2] E. G. Cristal and L. Young, "Theory and tables of optimum symmetrical TEM-mode coupled-transmission-line directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 544–558, Sept. 1965.
[3] R. Levy and L. F. Lind, "Synthesis of symmetrical branch-guide directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 80–89, Feb. 1968.
[4] R. Levy, "Zolotarev branch-guide couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 95–99, Feb. 1973.
[5] W. V. Tyminski and A. E. Hylas, "A wide-band hybrid ring for U.H.F.," *Proc. IRE*, vol. 41, pp. 81–87, Jan. 1953.
[6] A. F. Podell, "Some magic tees with 2 to 3 octaves bandwidth," in *Proc. 1969 G-MTT Int. Microwave Symp.*, pp. 317–319.
[7] J. W. Carr, "Balanced-line microwave hybrids," *Microwave J.*, vol. 16, no. 5, pp. 49–52, May 1973.
[8] D. Rubin and D. Saul, "mm Wave MICs use low value dielectric substrates," *Microwave J.*, vol. 19, no. 11, pp. 35–39, Nov. 1976.
[9] J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 246–252, Oct. 1956.
[10] S. Rehnmark, "TEM-mode power dividers and phase shifters—Antenna matching unit for the GEOS satellite," *Ericsson Technics*, vol. 31, no. 3, pp. 133–180, 1975.
[11] H. J. Riblet, "The application of a new class of equal-ripple functions to some familiar transmission-line problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 415–421, July 1964.
[12] S. J. Robinson, "Broad band hybrid junctions," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 671–672, Nov. 1960.
[13] R. J. Wenzel, "Exact design of TEM microwave networks using quarterwave lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 94–111, Jan. 1964.
[14] L. W. Chua, "New broadband matched hybrids for microwave integrated circuits," in *Proc. 2nd European Microwave Conf.* (Stockholm), section C 4/5, Aug. 1971.
[15] A. Matsumoto, *Basic Rational Sections for Microwave Filters.* Sapporo, Japan: Boneyido Printing Co., 1968, pp. 16–32.
[16] S. Rehnmark, "Theory and application of microwave couplers, phase shifters, and power dividers," Ph.D. dissertation, School of Electrical Engineering, Chalmers University of Technology, Göteborg, Sweden, Tech. Rep. 62, Apr. 1976, paper E, pp. 12, 13.